NUMERIC CALCULATION OF ANTIFERROMAGNETIC RESONANCE FREQUENCIES FOR THE NONCOLLINEAR ANTIFERROMAGNET

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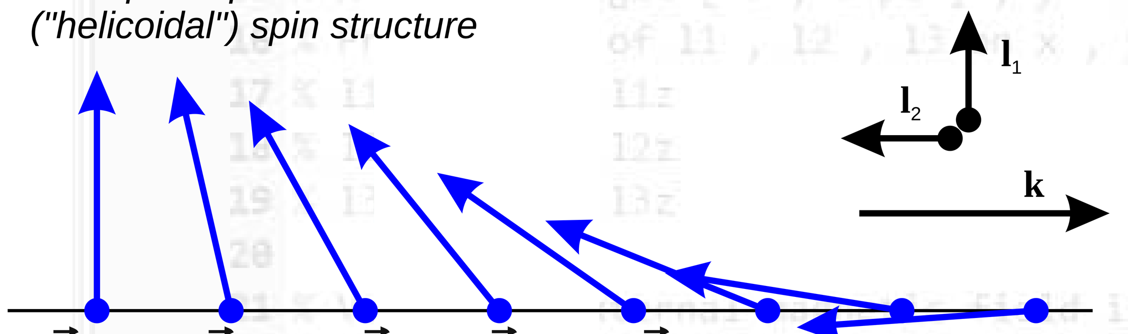
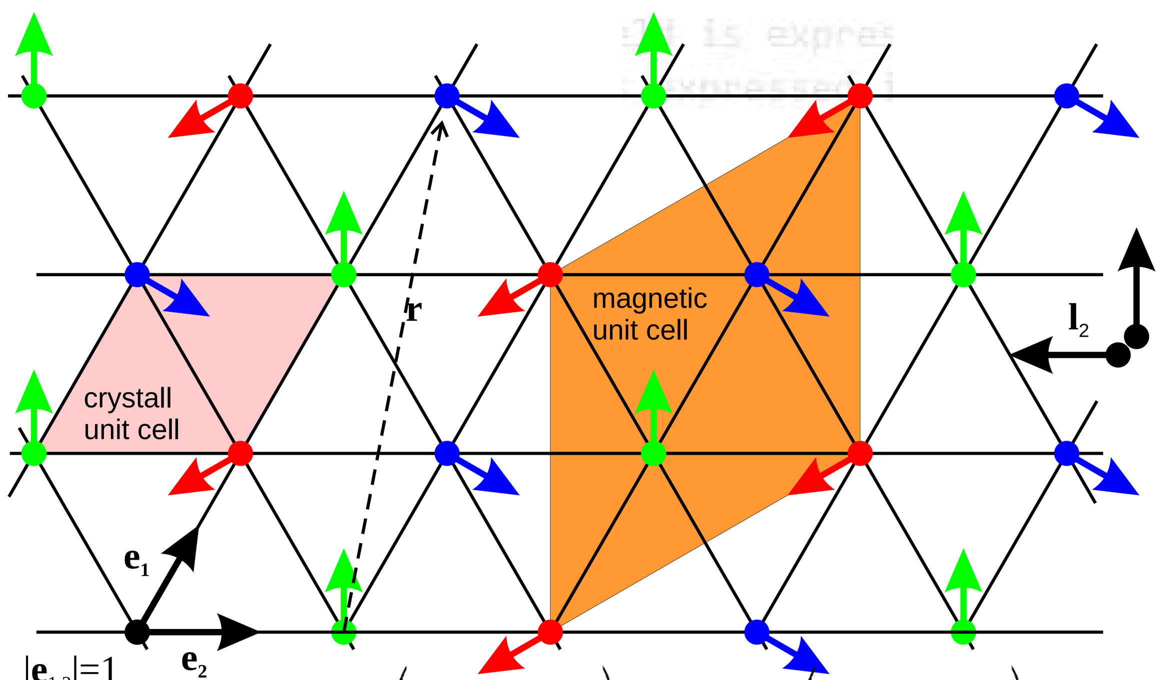
Example 1: parametrization of incommensurate ("helicoidal") spin structure



$$\vec{S}(\vec{r})=\vec{l}_1\cos(\vec{k}\vec{r})+\vec{l}_2\sin(\vec{k}\vec{r})$$

Example 2: parametrization of commensurate noncollinear (3 sublattices "triangular") structure



$$\vec{S}(r)=\vec{l}_1\cos\left(\frac{4\pi}{3a}\vec{r}\cdot\vec{e}_1\right)-\vec{l}_2\sin\left(\frac{4\pi}{3a}\vec{r}\cdot\vec{e}_1\right)$$

Problem:

Antiferromagnetic resonance (AFMR) experiment measures energies of k=0 spin waves. Its f(H) dependence contains information on order parameter structure and orientation. High energy resolution of AFMR (1 GHz, or 5μeV, is a routine!) makes it a very informative approach to study low-energy dynamics (especially zero-field gaps, low-energy modes, spin-reorientation...).

Interpretation of these data requires calculation of magnetic structure oscillations' eigenfrequencies.

We propose:

- flexible algorithm based on Andreev-Marchenko hydrodynamic approach
- implementation of this algorithm as Matlab or C++ codes and ready-to-use executable file available for download

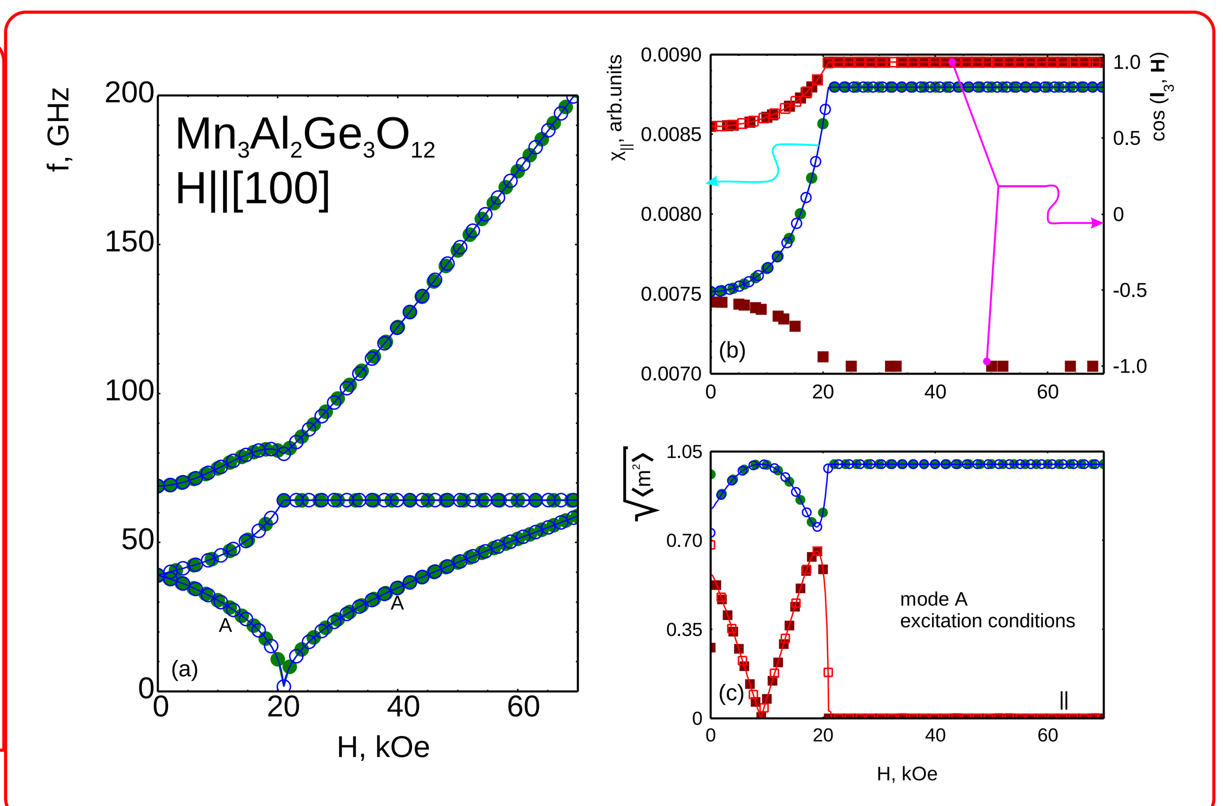
Setting model parameters (ini-file fragment)

```
[gamma:]
17.6
[l1:]
1.42e-5
[l2:]
1.42e-5
[l3:]
7.99e-6
[AnisotropyStart:]
l2zl2z;1
l1zl1z;-1
l1xl2x;1.15470054
l1yl2y;-1.15470054
[AnisotropyEnd:]
[Hdir:] (Semicolumn separated vector)
0;0;1
[Hstart:]
0
[Hstop:]
70
[Hstep:]
0.1
```

Setting model parameters. MatLab code

```
Hlow = 0;
Hhigh = 200;
delta = 1;

% test case of Mn3Al2Ge3O12
 gamma = 17.6;
 l1 = 1.42e-5; l2 = 1.42e-5; l3 = 7.99e-6;
 Uadd(l1x,l1y,l1z,l2x,l2y,l2z,l3x,l3y,l3z,h,hx,hy,hz) = 2/3^(1/2)*(l1x*l2x-l1y*l2y)+l1z^2-l2z^2;
```



Fragments of MatLab code and INI-file for C++ (and compiled EXE) with parameters set for 12-sublattices $Mn_3Al_2Ge_3O_{12}$; modeled AFMR f(H) and other output results.

Theoretical background (Andreev, Marchenko Sov. Phys. Usp. **130**, 39 (1980)):

0) Exchange interaction determines order parameter structure, all other interactions (anisotropy, field) are small corrections

1) Noncolinear antiferromagnetic order parameter can be parametrized by, maximum, three vectors $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$

2) Low energy dynamics can be described as oscillations of this vector field with the Lagrangian

$$L=\sum_i \frac{I_i}{2}\left(\dot{\vec{l}}_i+\gamma\left[\vec{l}_i\times\vec{H}\right]\right)^2-U_A\left(\{\vec{l}\}\right)$$

here $\chi_1=\gamma^2(I_2+I_3)$ etc., and U_A is the anisotropy energy (depends on order parameter structure)

Examples:

a) uniaxial $U_A=\beta\left(l_3^z\right)^2$

b) cubic garnet $U_A=\lambda\left[\left(l_2^z\right)^2-\left(l_1^z\right)^2+\frac{2}{\sqrt{3}}\left(l_1^x l_2^x-l_1^y l_2^y\right)\right]$

3) It is necessary to find equilibrium position and to solve set of the Euler-Lagrange equations in equilibrium vicinity.

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}_\alpha}-\frac{\partial L}{\partial\varphi_\alpha}=0$$

Mathematically simple, but cumbersome and quite probably not solvable analytically for arbitrary field direction

Input data (specified in an ini-file or explicitely in the code): $I_1, I_2, I_3, \gamma, \{\mathbf{H}\}$, functional form of $U_A(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3)$

Details:

1) Minimize "potential" energy over $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$ orientations (Euler angles)

$$\Pi=-\sum_i \frac{I_i}{2}\gamma^2\left[\vec{l}_i\times\vec{H}\right]^2+U_A\left(\{\vec{l}_i\}\right)$$

2) Replace Lagrangian by its quadratic expansion in the equilibrium vicinity

$$L=\sum_i \frac{I_i}{2}\left(\dot{\vec{l}}_i\right)^2+\gamma\sum_i I_i\left(\dot{\vec{l}}_i\left[\vec{l}_i\times\vec{H}\right]\right)-\frac{1}{2}\sum_{\beta,\delta}\left(\frac{\partial^2\Pi}{\partial\varphi_\beta\partial\varphi_\delta}\right)_0\varphi_\beta\varphi_\delta$$

3) Substitute small oscillations to Euler-Lagrange equations and reduce problem to degeneracy of linear equations det M=0

$$M_{\alpha\beta}=-\omega^2\sum_i I_i\left(\left(\frac{\partial\vec{l}_i}{\partial\varphi_\alpha}\right)_0\cdot\left(\frac{\partial\vec{l}_i}{\partial\varphi_\alpha}\right)_0\right)+2i\omega\gamma\sum_i\left(\left(\frac{\partial\vec{l}_i}{\partial\varphi_\alpha}\right)_0\cdot\left[\left(\frac{\partial\vec{l}_i}{\partial\varphi_\beta}\right)_0\times\vec{H}\right]\right)+\left(\frac{\partial^2\Pi}{\partial\varphi_\alpha\partial\varphi_\beta}\right)_0$$

4) Codes tested against analytically solvable cases

Output data: f(H) dependences, order parameter orientation, static magnetization components, excitation conditions (oscillating magnetization)

References and Downloads:

Applied Magnetic Resonance **47**, 1069 (2016); arXiv: 1606.09349

Source files (C++, Matlab), compiled Win32 executable, examples of ini-files for executable are available for free download at: www.kapitza.ras.ru/rgroups/esrgroup/numa.html