different field components are connected by the relation

$$\phi_q = \phi_0 + |q| (\phi_{-1} - \phi_0) \quad (3)$$

where $\phi_0 = \psi_0$ and ϕ_{-1} is arbitrary. Relation (3) denotes the synchronization of the SRR components, such that the total output radiation from the resonator constitutes a periodic sequence of ultrashort pulses; this sequence has an arbitrary initial phase given by $\phi_q = \phi_0 + |\vec{q}| .(\phi_{-1} - \phi_0) + m_q\pi$ (m_q is a definite integer). It signifies that the entire sets of SRR components is broken up into two groups, in each of which a phase relation of the type (3) is satisfied.

[1] V. N. Lugovoi, Opt. spektr. 27, No. 4 (1969).
[2] V. N. Lugovoi, ibid. 27, No. 5 (1969).

INTERNAL GRAVITATIONAL WAVES IN A SUPERFLUID SOLUTION

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Submitted 5 November 1969
ZhETF Pis. Red. 10, No. 11, 567 - 570 (5 December 1969)

It is known that internal gravitational waves can propagate in ordinary liquids as a result of inhomogeneity of the liquid in the gravitational field [1]. For such waves to exist it is necessary that the thermal equilibrium be established much more slowly than the mechanical equilibrium. It is clear that no such phenomenon can exist in superfluid helium, in which this requirement is not satisfied; any disturbance of the equilibrium can lead here only to waves of first or second sound.

A superfluid solution is analogous in a certain sense to an ordinary liquid, in that a temperature gradient can be produced in it under stationary conditions [2]. We shall show below that gravitational waves can also exist in such a solution.

The conditions for mechanical equilibrium of a superfluid solution in a gravitational waves are

$$\nabla p - \rho \mathbf{g} = 0; \qquad \nabla \mu_4 - \mathbf{g} = 0$$

where p is the pressure, ρ the density, g the free-fall acceleration, and μ_4 the chemical potential of He^4 in the solution.

In a gravitational wave, p and μ_4 differ little from their equilibrium values. This means that p and μ_4 should be regarded as constant when the thermodynamic quantities are differentiated. We assume the temperature gradient to be small enough, in order for the changes of the equilibrium values of the thermodynamic quantities over distances on the order of a wavelength to be small.

The hydrodynamic equations of solutions [2] linearized for the purpose of our problem can be written in the form

$$\frac{\partial \rho'}{\partial t} + \operatorname{div} \mathbf{j}' = 0, \quad (1)$$

$$\rho_n \frac{\partial v_n'}{\partial t} + \rho_s \frac{\partial v_s'}{\partial t} + \nabla \rho' - \rho' g =$$

$$= \eta \Delta v_n' + \zeta_1 \nabla \operatorname{div} j' + \left(\frac{1}{3}\eta - \zeta_1 \rho + \zeta_2\right) \nabla \operatorname{div} v_n' , \tag{2}$$

$$\frac{\partial v_s'}{\partial t} + \nabla \mu_4' = \zeta_3 \nabla \operatorname{div} j' + (\zeta_4 - \rho \zeta_3) \nabla \operatorname{div} v_n' , \tag{3}$$

$$\frac{\partial}{\partial t}(\rho c)' + \nabla(\rho c) v_n' + \rho c \operatorname{div} v_n' = \rho D(\Delta c' + k_T T^{-1} \Delta T'), \tag{4}$$

$$\frac{\partial S'}{\partial t} + \nabla S v_n' + S \operatorname{div} v_n' =$$

$$= \kappa_{eff} T^{-1} \Delta T' + c^{-1} S D(\Delta c' + k_T T^{-1} \Delta T') . \tag{5}$$

(All the symbols in (1) - (5) are the same as in [2]; the primed quantities denote small additions to the equilibrium values.) We have retained the terms with ∇S and $\nabla(\rho c)$ in the last two equations, since it will be shown that they are the ones determining the phenomenon.

Using the fact that v_s is potential, we obtain from (2) after some transformations

$$\left(\rho_n \frac{\partial}{\partial t} - \eta \Delta\right)(\Delta v_n' - \nabla \operatorname{div} v_n') = g \Delta \rho' - \nabla(g \nabla \rho') , \tag{6}$$

and from (4) and (5)

$$\frac{\partial s'}{\partial t} + \nabla s v_n' = \kappa_{eff} T^{-1} \Delta T' (\rho c)^{-1}; \quad s = \frac{S}{\rho c} . \tag{7}$$

If we neglect the term with ∇s in (7), then we obtain only an ordinary damped temperature wave. It is therefore clear that all the terms in the last equation are in general of the same order of magnitude. Returning to the initial system we see that in this case we should have, generally speaking,

$$\nabla S v_n' \approx S \operatorname{div} v_n' .$$

We seek a solution in the form of a plane wave $v_n' \sim \exp(-i\omega t + i\vec{k}\cdot\vec{r})$. Then, in view of the smallness of ∇S, we have

$$k^2 v_n' >> k \operatorname{div} v_n'$$

i.e., the oscillations in question are transverse, just as in an ordinary liquid. Neglecting the small term in (6) we obtain from the condition for the compatibility of (6) and (7) the sought connection between ω and $\vec{k}$:

$$(\omega + i \chi k^2)(\omega + i \nu k^2) - \omega_0^2 = 0, \tag{8}$$

$$\chi = (\rho c T)^{-1} \kappa_{\text{eff}} \left(\frac{\partial T}{\partial s}\right)_{p,\mu_4}; \qquad \nu = \rho_n^{-1} \eta,$$

$$\omega_0^2 = -g\rho_n^{-1}\left(\frac{\partial \rho}{\partial s}\right)_{p,\mu_4} \frac{\partial s}{dz} \sin\theta.$$

Here θ is the angle between the direction of the z axis (vertically upward) and the direction of $\vec{k}$. Substituting this solution into the exact system of equations [2], we can verify that all terms neglected by us are indeed small.

At small $\vec{k}$ and at $\omega_0^2 > 0$, Eq. (8) described weakly-damped oscillations:

$$\omega = \omega_0 - \frac{i}{2}(\chi + \nu) k^2.$$

In the opposite case of large $\vec{k}$ we have two damped waves, viscosity and temperature. Finally, if the condition

$$\omega_0^2 + \chi\nu k^4 > 0 \tag{9}$$

is violated, then (8) has a solution with a positive imaginary part, i.e., the equilibrium state is unstable against perturbations of this type. The condition (9) is thus a condition for the absence of convection in a superfluid solution, and the quantities

$$\rho c T \kappa_{\text{eff}}^{-1} \rho_n^{-1} \eta \left(\frac{\partial s}{\partial T}\right)_{p,\mu_4} \text{ and } g\rho_n \eta^{-2}\left(\frac{\partial \rho}{\partial s}\right)_{p,\mu_4} \frac{sp}{dz} \ell^4$$

(where ℓ is the characteristic dimension) play the role of the Prandtl and Grashof numbers, respectively.

I take the opportunity to thank A. F. Andreev for constant interest and numerous valuable hints.

[1] L. D. Landau and E. M. Lifshitz, Mechanika sploshnykh sred, 1953, Sec. 14 [Fluid Mechanics, Addison-Wesley, 1958].

[2] I. M. Khalatnikov, Vvedenie v teoriyu sverkhtekuchesti (Introduction to the Theory of Superfluidity), 1965, Sec. 24. [Benjamin, 1965].