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Low Temperature Mobility of Steps on the ^{4}He Crystal Facets: Effects of ^{3}He Impurities

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At low temperatures, the dissipation due to scattering of thermal phonons by elementary steps on the facets of helium crystals is very small; ^{3}He impurities may dominate in dissipative processes even at low concentrations. We calculate the step mobility determined by collisions of the step with impurities in bulk liquid and impurities absorbed on the liquid-solid interface, in different temperature and concentration regions.

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1. INTRODUCTION

Experiments on the spiral growth of helium crystals at ultralow temperatures provide an excellent opportunity to study dynamic properties and the structure of individual elementary steps on the solid-superfluid helium interface[1] . These properties are of great interest owing to the essentially quantum nature of the step motion[2] .

Kinetic characteristics of steps in pure ^{4}He have been measured[1,3] and discussed theoretically[4,5] . In the limit of low temperatures and small driving forces, where only thermal phonons contribute to the total dissipation, the step mobility μ is independent on the driving force and tends to infinity as T^{-3}. Under these conditions, the mobility may be very sensitive to the presence of impurities.

When discussing possible effects of ^{3}He impurities on the step mobility, one may neglect impurities dissolved in the solid phase, because at low temperatures, well below 0.3K, their concentration is very small. Instead, one should take into account absorption of ^{3}He on the liquid-solid interface. According to[6,7] , absorbed ^{3}He behaves as a 2D Fermi gas of quasiparticles with effective mass $m_2 \approx 2.3m$ (m is the bare mass of ^{3}He atoms), saturation density $n_{sat} \approx 2.5 \times 10^{14}\text{cm}^{-2}$, the band width $\Delta \approx 0.7$K, and the binding energy at the interface $\epsilon_s \approx 4$K. In this paper we consider elastic collisions of both types of impurities, 3D (in the bulk liquid) and 2D (on the interface), with a moving step, and their contributions to the total friction. We also briefly discuss possible inelastic processes.

2. ELASTIC SCATTERING OF 3D IMPURITIES

The mobility μ is the ratio of the step velocity v to the friction force F per unit length of the step. It is convenient to calculate this friction in the reference frame moving with the step. In the following, we assume that the liquid is situated at $z > 0$, the step is parallel to y axis, and x is the direction of their motion. The specific friction (resistance) is:

$$\frac{1}{\mu} = \frac{F}{v} = \frac{1}{v}\int Q(\vec{k})n(\vec{k})d^3k, \tag{1}$$

where n is the impurities distribution function (in the moving frame) and Q is the average x-component of the momentum transferred per unit time at collisions of ^{3}He particles (with a given wave vector $\vec{k}$) with the step:

$$Q = \frac{\hbar^2}{m_3}q\int (k_x - k_x')d\sigma, \tag{2}$$

where k_x and k_x' are x-components of wavevectors of incident and scattered particles respectively, m_3 is their effective mass, $q = \sqrt{k^2 - k_y^2}$ and σ is the scattering cross section.

The value of Q, generally speaking, depends on the step height a and on its structure on the microscopic scale. We calculate Q in the long-wave limit, $ka \ll 1$, in the first approximation of standard diffraction theory. Consider first the case of very low temperature and low density of impurities n_3, when their characteristic wavelength is even larger than the step width, and the step structure is not important. In this limit the ^{3}He-step scattering amplitude can be written as

$$f = a\sqrt{\frac{2}{\pi i q}}\,\frac{k_z k_z'}{k_x - k_x'}. \tag{3}$$

From Eqs. (1), (2) and (3) we obtain for nondegenerate and degenerate ^{3}He gas respectively (T_{F3} is the Fermi temperature):

$$\frac{1}{\mu} = \frac{a^2 m_3 T}{\hbar}n_3, \qquad T \gg T_{F3} \tag{4}$$

and

$$\frac{1}{\mu} = \frac{(3\pi^2)^{2/3}}{5}\hbar a^2 n_3^{5/3}, \qquad T \ll T_{F3}. \tag{5}$$

Now turn to the case of higher temperatures (or higher densities), when the impurity wavelength becomes on the order of the step width. Here we should take into account high frequency, zero-point oscillations of the step. In fact, in the presence of high-frequency oscillations an incident particle see an average profile of the step. The amplitude of such oscillations ξ_0 is much larger than the correlation length $\xi \approx 2a$ in the "weak coupling" approximation[3] . Indeed, we may regard the step as a quantum string with linear tension β and the oscillation spectrum[8]

$$\omega^2 = \frac{\pi\beta\rho_l}{a^2(\rho_s - \rho_l)^2}\,\frac{k^2}{ln(1/k\xi)} = (c_s k)^2\,, \tag{6}$$

where ρ_s and ρ_l are the densities of solid and liquid helium respectively. In the harmonic approximation the mean square displacement of such a step is

$$<(\Delta x)^2> = \xi_0^2 \approx \frac{\hbar}{a(\rho_s - \rho_l)}\sqrt{\frac{\rho_l}{\pi\beta}\,ln(\frac{\theta_D}{T})}\,, \tag{7}$$

where θ_D is the characteristic Debye temperature.

In Eq. (7) only high-frequency oscillations ($\theta_D > \hbar\omega > T$) are taken into account. The value of ξ_0 depends very weakly on the parameters of this cut-off. Numerically, at $T \sim 10$mK, $\xi_0 \approx 3$nm $\approx 5\xi$. We conclude that the effective width of the step is determined mainly by zero-point oscillations. In the same approximation, the step effective profile looks as follows:

$$\zeta(x) = \frac{a}{\sqrt{2\pi\xi_0^2}}\int_{-\infty}^{x}\exp(-\frac{x'^2}{2\xi_0^2})dx' \tag{8}$$

We have now instead of Eq. (3)

$$f = a\sqrt{\frac{2}{\pi i q}}\,\frac{k_z k_z'}{k_x - k_x'}\,\exp\left(-\frac{\xi_0^2}{2}(k_x - k_x')^2\right). \tag{9}$$

For higher temperatures, $T \gg \hbar^2/m_3\xi_0^2$ (and low densities, $T_{F3} \ll T$,), Eq. (9) yields

$$\frac{1}{\mu} = \frac{\sqrt{2}}{\pi}\,\frac{a^2\sqrt{m_3 T}}{\xi_0}n_3\,, \tag{10}$$

while for $T \ll \hbar^2/m_3\xi_0^2$ we return to Eq. (4). In the opposite case of higher densities, $n_3\xi_0^3 \gg 1$ (and low temperatures, $T \ll T_{F3}$), we have

$$\frac{1}{\mu} = \frac{3^{4/3}}{8\pi^{1/6}}\,\frac{\hbar a^2 n_3^{4/3}}{\xi_0}\,. \tag{11}$$

3. ELASTIC COLLISIONS WITH 2D IMPURITIES

For ^{3}He impurities absorbed on the interface, the step plays the role of a potential well $U(x)$, uniform in y direction. According to[7] , the depth of this well can be estimated as 9mK, with the well width 4ξ. Taking into account large value of ξ_0 and the step effective profile described by Eq.(8), we should assume the following form of the effective potential:

$$U(x) = -U_0 \exp(-x^2/2\xi_0^2), \tag{12}$$

with $U_0 \approx 3$mK.

Owing to rather large value of ξ_0, this well turns out to be sufficiently deep for ^{3}He atoms: $U_0 \sim \hbar^2/m_2\xi_0^2$. The coefficient of elastic reflection of ^{3}He quasiparticles from such a well R is a function of the x component of their momentum. This function can not be calculated using the perturbation theory; $R \approx 1$ at $k_x\xi_0 \leq 1$ and is exponentially small at $k_x\xi_0 \gg 1$ (see[9]). In the following, we assume for simplicity $R = 1$ at $k_x\xi_0 < 1$ and $R = 0$ at $k_x\xi_0 > 1$. We neglect also the difference between real (unknown) spectrum of the 2D quasiparticles and simple quadratic spectrum with constant mass. Such an approximation seems to be sufficiently accurate in two different limits: almost empty, or almost saturated band. In the latter case instead of ^{3}He quasiparticles we should speak about holes. For the interval between these limits our approximation should be regarded just as a simple interpolation.

Elastic collisions of 2D impurities (or holes) with moving steps result in additional resistance. Instead of Eqs.(1),(2), we have to write in this case

$$\frac{1}{\mu} = \frac{2\hbar^2}{m_2 v} \int |k_x| k_x n(\vec{k}) R(k_x) d^2k, \tag{13}$$

where $n(\vec{k})$ is now the distribution function of 2D impurities, again in the moving frame.

Further calculations are straightforward. In the limit of low densities ($n_2\xi_0^2 \ll 1$) and temperatures($T \ll \hbar^2/m_2\xi_0^2$) we obtain

$$\frac{1}{\mu} = \sqrt{\frac{32}{\pi} m_2 T}\; n_2, \qquad T \gg T_{F2} \tag{14}$$

and

$$\frac{1}{\mu} = \frac{32}{3\sqrt{2\pi}}\, \hbar\, n_2^{3/2}, \qquad T \ll T_{F2}, \tag{15}$$

where T_{F2} is the Fermi temperature of the 2D impurities. At higher temperatures (and still low densities) the result is

$$\frac{1}{\mu} = \frac{\hbar^4}{\sqrt{2\pi}} \frac{n_2}{\xi_0^4 (m_2 T)^{3/2}}\,, \tag{16}$$

while at higher densities ($n_2\xi_0^2 \gg 1$, but of course $n_2 < n_{sat}/2$), we have

$$\frac{1}{\mu} = \frac{\hbar}{\sqrt{2}\,\pi^{5/2}\xi_0^4} \frac{1}{\sqrt{n_2}}\,, \tag{17}$$

which should be regarded as rather crude estimate (see above).

4. INELASTIC PROCESSES

Generally speaking, scattering of impurities by a step may be accompanied by emission and absorption of quanta of the step oscillations (Eq.6). The cross section of such inelastic process is determined by the impurity-step interaction. The main term in this interaction may be written in the form

$$\hat{H}_{int} = -\frac{1}{2}(\hat{\vec{p}}\,\hat{\vec{v}}_s + \hat{\vec{v}}_s\hat{\vec{p}}), \tag{18}$$

where $\hat{\vec{p}}$ is the operator of momentum of a ^{3}He particle and $\hat{\vec{v}}_s$ is the operator of superfluid velocity at a given point in the vicinity of oscillating step.

In the first Born approximation this Hamiltonian yields

$$\sigma_{inel} \sim a^4 k^3 \frac{\hbar}{m c_s a} \sim a^4 k^3. \tag{19}$$

This result should be compared with the values of average elastic scattering cross sections, following from Eq.(3) and Eq.(9), $\sigma_{el} \sim a^2 k$ and $\sigma_{el} \sim a^2/\xi_0$ respectively. Similar situation takes place in 2D case, where $\sigma_{el} \sim a < R >$. We conclude that inelastic scattering results in small corrections to the step mobility, which should be neglected within our accuracy.

One more type of inelastic processes, "sweep off" of 2D impurities, becomes possible at sufficiently high step speeds. The minimum critical velocity for this process is rather high,

$$v_c = \sqrt{\frac{\epsilon_s - \Delta}{m_3}} \approx 60\text{m/sec} \tag{20}$$

(here we have assumed for simplicity the mass of 2D holes equal to m_3). At $v > v_c$, the step mobility significantly decreases provided that 2D band is saturated and n_3 is small. However, at such high speeds even in absence of impurities other nonlinear effects are expected[1,5] , which can disguise the effect of "sweep off".

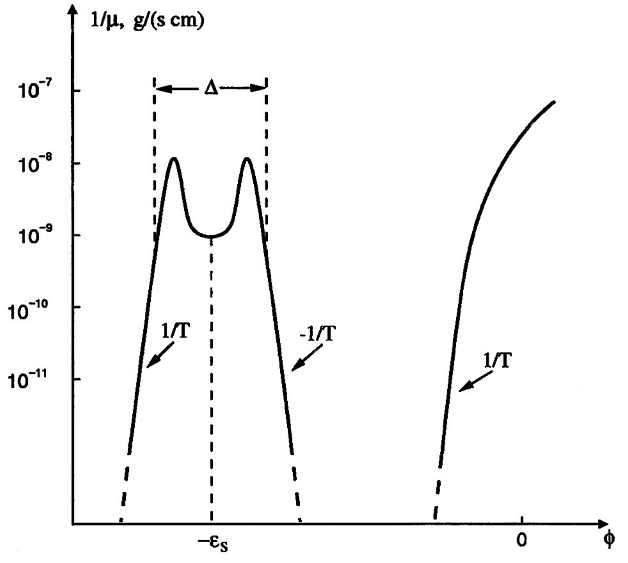

Fig. 1. Schematic plot of the impurities contribution to the resistance $1/\mu$ *versus* ϕ for a constant T.

5. DISCUSSION

In real experimental situation, both types of impurities, 2D and 3D, contribute to the total resistance. However, owing to rather large value of ϵ_s, at low temperatures actually only one of them can be active at a given chemical potential of the impurities ϕ. At low values of ϕ, in the range of energies of 2D band, when the contribution of 2D impurities is large, the equilibrium density of bulk impurities is exponentially small, $n_3 \sim \exp(-\epsilon_s/T)$, and their contribution is negligible. With increasing ϕ, the 2D band becomes saturated, and both contributions turn out to be small. At further increase of ϕ, the resistance due to bulk ^{3}He increases in proportion to n_3 (see Fig. 1).

To clarify the significance of the 2D and 3D ^{3}He contributions compared to the phonon one, imagine a cell of 1cm^3 with an interface of 1cm^2. Assuming that there is no absorption of the impurities on the cell walls, we can calculate the total resistance for a given number N of ^{3}He atoms in the cell at different temperatures. Fig. 2 shows the smoothed results of such calculations for N from 10^{10} to 10^{19} (the latter case corresponds to the bulk concentration $X_3 \approx 400$ppm).

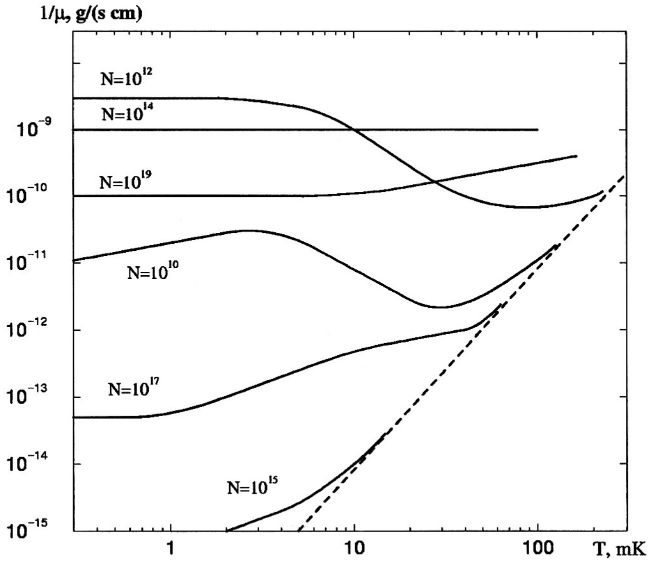

Fig. 2. The total resistance $1/\mu$ as a function of temperature for different N. The dashed line is the phonon contribution according to Ref. 4.

The contribution of the 2D impurities dominates at $N \leq 2 \cdot 10^{14}$ (at $N = 10^{14}$ the 2D band is almost half-filled); note that numerically it may be even larger than the 3D contribution at X_3 up to 10^3ppm. On the contrary, at $N \geq 10^{15}$ we may neglect the effect of the 2D ^{3}He. Such a situation is favourable from the experimental point of view, since it allows to study these two contributions separately. However, possible trapping of ^{3}He on the cell walls, or on vortices, may create serious difficulties in the determination of the remaining amount of the impurities, especially at small values of N (see, for example,[7]). Therefore, it is important to take care of accurate control of the ^{3}He chemical potential in the cell, which is not very easy to do in a real experiment.

As one can see also from Fig. 2, the effect of impurities may exceed the phonon resistance at temperatures below 0.1K in a wide range of concentrations. It could be observed in experiments on spiral growth of helium facets, like[1] , or in measurements of low temperature mobility of vicinal surfaces with very low density of steps, like[3] . Such experimental data would give valuable information on the microscopic structure of elementary steps and their interaction with impurities. Of special interest is the possibility to study dynamic properties of the 2D ^{3}He gas on the helium crystal facet, which may be regarded as a perfect substrate.

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